Liquidity saving mechanisms in collateral-based RTGS payment systems[1]

Marius Jurgilas Antoine Martin

Bank of England

Federal Reserve Bank of New York

[1] The views expressed herein are those of the authors and do not necessarily reflect the views of the Bank of England, the Federal Reserve Bank of New York, or the Federal Reserve System.

Payment systems

- Role of payment systems
- Evolution of payment systems:
  from DNS to RTGS to "enhanced" RTGS
- Two (three) types of RTGS:
  - fee-based intraday credit
  - collateral-based intraday credit
  - collateral-pool-based

Objective of the study

Policy question:

Should liquidity saving mechanisms (LSMs) be introduced in CHAPS?

Literature

- **Delay cost:**
  Angelini (1998, 2000), Bech and Garratt (2003)
- **Settlement risk:**
  Mills and Nesmith (2008), Nellen (2009), Jurgilas and Ota (2010)
- **Simulations:**
  Galbiati and Soramäki (2009), Denbee and Norman (2010)
- **LSMs in fee-based systems:**
  Martin and McAndrews (2008), Atalay et al. (2008)

2 contributions of this paper:

1. Characterization of equilibrium and social planner allocations in collateral-based RTGS with/without LSMs
2. Welfare implications of introducing LSMs

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2 contributions of this paper:

1. Characterization of equilibrium and social planner allocations in collateral-based RTGS with/without LSMs
2. Welfare implications of introducing LSMs

Key results

- Equilibrium and planner's allocation can differ in collateral-based RTGS without LSM:
  - Too much delay in equilibrium if equilibrium allocation $\not\equiv$ planner's.
- For some parameters equilibrium allocation $\equiv$ planner's:
  - All banks delay if collateral cost is high (in equilibrium and planner's allocation).
- LSMs *always* welfare improving in collateral-based RTGS, in contrast to Atalay et al. (2008):
  - BoE is implementing queueing algorithm in CHAPS.

Main assumptions

- Agents:
  - Infinitely many identical banks
  - nonoptimizing settlement system
- Payments:
  - Liquidity shocks (payments to/from settlement systems, cannot be delayed)
  - Urgent payments (delay cost γ if delayed)
  - Non-urgent payments (can be delayed without any cost)
  - Payments between banks form offsetting cycles
- There is a cost if a payment submitted for settlement does not settle.
- Posting collateral early is cheaper.

Structure

Liquidity shock of size $1 - \mu$:

- $\lambda = -1$ with prob. $\overline{\pi}$
- $\lambda = 1$ with prob. $\overline{\pi}$
- $\lambda = 0$ with prob. $1 - 2\overline{\pi}$

Fraction μ of payments are:

- **urgent**, with prob θ
- **non-urgent**, with prob. $1 - \theta$

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Timing

0 Choose the amount of collateral to be posted, L_0

1 Observe liquidity shock λ and liquidity in the morning:

$$L_1 = L_0 + \lambda(1 - \mu)$$

1 Observe the type of payment μ to be made (γ=0 or $\gamma > 0$)

2 Submit a payment $P = 1$ or delay $P = 0$ until the afternoon

2 With LSM decide if to queue $Q = 1$ or not $Q = 0$

3 Incoming payments observed

4 Post additional collateral at the end of the day if needed.

Settlement

Settlement

A payment of μ *submitted* for settlement settles if:

- $L_1 \geq \mu$
- $0 < L_1 \leq \mu$ **and** a payment is received from the other bank

A *queued* payment settles if an incoming payment is received.

Otherwise payment does not settle.

Cost of settlement:

- If a payment is submitted, but does not settle, a bank incurs a delay cost γ and an additional cost $R \geq 0$.
- γ only if payment is not submitted, or queued and not settled.

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Probability to receive a payment in the morning

ω^i if you don't submit **or** submit without sufficient liquidity.

ω^s if you submit **and** you have sufficient liquidity.

ω^q if you queue.

Parameter restrictions:

- Liquidity shocks are small: $\mu \geq \frac{2}{3}$
- Relatively small cost of collateral in the morning: $\overline{\pi}\Psi \geq \kappa$

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Problem solved

$$\min_{L_0} \underset{\lambda,\gamma}{E} \left[\min_{P} \underset{\phi(\omega)}{E} (C_1 + C_2) \right]$$

s.t.

$$C_1 = \kappa L_0 + PI(L_1 < \mu)(1-\omega^i)(R+\gamma) + (1-P)\gamma$$

$$C_2 = [(1-P)(1-\omega^i) + PI(L_1 < \mu)(1-\omega^i)] \max\{\mu - L_1, 0\}\Gamma$$

Equilibrium

A strategy $\{L_0^*, P^*(\lambda,\gamma;L_0)\}$ is a symmetric subgame perfect Nash equilibrium strategy, if there exists a set of beliefs $\omega = \{\omega^s, \omega^i\}$ such that:

$$\begin{aligned} P^*(\lambda,\gamma;L_0) &= \arg \min_{P(\lambda,\gamma;L_0)} C(L_0, P(\lambda,\gamma;L_0),\omega)\ \forall\, \lambda,\gamma,L_0 \\ L_0^* &= \arg\min_{L_0} \underset{\lambda,\gamma}{E} \left[C(L_0, P^*(\lambda,\gamma;L_0),\omega)\right] \\ \omega &= \Omega(L_0^*,\omega) \end{aligned}$$

L_0^* and $P^*(\lambda, \gamma; L_0)$

L3 Any value of L_0 different from $L_0 \in \{1-\mu, 2\mu-1, \mu, 1\}$ cannot support an equilibrium.

P4 All banks submit payments early if $L_1 \geq \mu$.

P5 Banks with insufficient collateral, $L_1 < \mu$, and an urgent payment delay if $(1-\omega^i)(R+\gamma) > \gamma$.

L6 In equilibrium, $L_0 < 1$ and $\omega^i < 1$.

P7 If $L_1 < \mu$ banks with an non-urgent payment delay.

$\Omega(L_0^*, \omega)$

Fraction of banks:

- τ_d: $P = 0$
- τ_s: $P = 1$ and $L_1 \geq \mu$
- τ_i: $P = 1$ and $L_1 \leq \mu$

$\tau_d + \tau_s + \tau_i = 1.$

$n = 2$: $\Omega^s = \tau_s + \tau_i$

$n = 3$: $\Omega^s = \tau_s + \tau_i(\tau_i + \tau_s)$

n: $\Omega^s = \tau_i^{n-1} + \sum_{k=0}^{n-2} \tau_s \tau_i^k$

$n \to \infty$: $\Omega^s = \frac{\tau_s}{\tau_s + \tau_d}$

$$\Gamma = \frac{(1-\tau_s)^{n-1}}{n} \Psi < \Psi$$

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Equilibrium payment strategy $P^*(\lambda, \gamma, L_0^*)$

$L_0^* = \mu$

If parameters are such that $L_0^* = \mu$ and $1 - \overline{\pi} \leq \frac{R}{R+\gamma} \leq \frac{1-\overline{\pi}}{1-\overline{\pi}\theta}$, then multiple equilibria in payment behavior are possible:

(i) $\omega^i = \frac{1-\overline{\pi}}{1-\overline{\pi}\theta}$, and

$$P^*(\lambda, \gamma, L_0^*) = \begin{cases} 1, & \text{if } \lambda = 0, 1; \text{ or } \lambda = -1 \text{ and } \gamma > 0 \\ 0, & \text{if } \lambda = -1 \text{ and } \gamma = 0. \end{cases}$$

(ii) $\omega^i = 1 - \overline{\pi}$, and $P^*(\lambda, \gamma, L_0^*) = \begin{cases} 1, & \text{if } \lambda = 0, 1; \\ 0, & \text{if } \lambda = -1. \end{cases}$

(i) is the unique equilibrium, if $1 - \overline{\pi} > \frac{R}{R+\gamma}$, while (ii) is the unique equilibrium if $\frac{R}{R+\gamma} > \frac{1-\overline{\pi}}{1-\overline{\pi}\theta}$.

Equilibrium payment strategy $P^*(\lambda, \gamma, L_0^*)$

$L_0^* = 2\mu - 1$

If parameters are such that $L_0^* = 2\mu - 1$ and $\overline{\pi} \leq \frac{R}{R+\gamma} \leq \frac{\overline{\pi}}{1-(1-\overline{\pi})\theta}$, then multiple equilibria in payment behavior are possible:

(i) $\omega^i = \frac{1-\overline{\pi}}{1-\overline{\pi}\theta}$, and

$$P^*(\lambda, \gamma, L_0^*) = \begin{cases} 1, & \text{if } \lambda = 1; \text{ or } \lambda = -1, 0 \text{ and } \gamma > 0 \\ 0, & \text{if } \lambda = -1, 0 \text{ and } \gamma = 0. \end{cases}$$

(ii) $\omega^i = 1 - \overline{\pi}$, and $P^*(\lambda, \gamma, L_0^*) = \begin{cases} 1, & \text{if } \lambda = 1; \\ 0, & \text{if } \lambda = -1, 0. \end{cases}$

(i) is the unique equilibrium, if $\overline{\pi} > \frac{R}{R+\gamma}$, while (ii) is the unique equilibrium if $\frac{R}{R+\gamma} > \frac{\overline{\pi}}{1-(1-\overline{\pi})\theta}$.

Equilibrium payment strategy $P^*(\lambda, \gamma, L_0^*)$

$L_0^* = 1 - \mu$

If parameters are such that $L_0^* = 1 - \mu$, then the unique payment equilibrium is characterized by: $\omega^i = 0$, and $P^*(\lambda, \gamma, L_0^*) = 0$.

Optimal collateral choice

$\frac{R}{R+\gamma} > \frac{\overline{\pi}}{1-(1-\overline{\pi})\theta}$

If $\frac{R}{R+\gamma} > \frac{\overline{\pi}}{1-(1-\overline{\pi})\theta}$ a subgame perfect Nash equilibrium strategy is:

(i) $L_0^* = \mu$, $\omega^i = 1-\overline{\pi}$, $P^*(\lambda,\gamma,L_0^*) = \begin{cases} 1, & \text{if } \lambda = 0, 1; \\ 0, & \text{if } \lambda = -1. \end{cases}$
if $(1-\mu)\kappa < \gamma\theta(1-2\overline{\pi})$ and $(2\mu-1)\kappa < \gamma\theta(1-\overline{\pi})$.

(ii) $L_0^* = 2\mu - 1$, $\omega^i = 1-\overline{\pi}$, $P^*(\lambda,\gamma,L_0^*) = \begin{cases} 1, & \text{if } \lambda = 1; \\ 0, & \text{if } \lambda = -1, 0. \end{cases}$
if $(1-\mu)\kappa > \gamma\theta(1-2\overline{\pi})$ and $(3\mu-2)\kappa < \overline{\pi}\gamma\theta$.

(iii) $L_0^* = 1-\mu$, $\omega^i = 0$, and $P^*(\lambda,\gamma,L_0^*) = 0$.
if $(3\mu-2)\kappa > \overline{\pi}\gamma\theta$ and $(2\mu-1)\kappa > \gamma\theta(1-\overline{\pi})$.

Welfare

Let $W(L_0 = x)$ denote the welfare associated with $L_0 = x$:

$$\begin{aligned} W(L_0=\mu) > W(L_0=2\mu-1) &\Leftrightarrow (1-2\overline{\pi})\theta\gamma > (1-\mu)\kappa, \\ W(L_0=2\mu-1) > W(L_0=1-\mu) &\Leftrightarrow \overline{\pi}\theta\gamma > (3\mu-2)\kappa, \\ W(L_0=\mu) > W(L_0=1-\mu) &\Leftrightarrow (1-\overline{\pi})\theta\gamma > (2\mu-1)\kappa. \end{aligned}$$

Intuition:

- $\kappa \uparrow \Rightarrow L_0 \downarrow$
- $\gamma\theta \uparrow \Rightarrow L_0 \uparrow$
- $\overline{\pi} \uparrow \rightarrow$: $1-\mu \succ \mu$
  but also $2\mu-1 \succ 1-\mu$ and $2\mu-1 \succ \mu$

Fee-based vs collateral-based RTGS

Fee-based:

- Strategic interaction ⇒ multiple equilibria
- Up to 4 equilibria

Collateral-based:

- Banks with sufficient liquidity submit
- Unique equilibrium with short cycles, 2 equilibria with long
- Multiplicity due to banks with insufficient funds and urgent payments

Introducing LSM

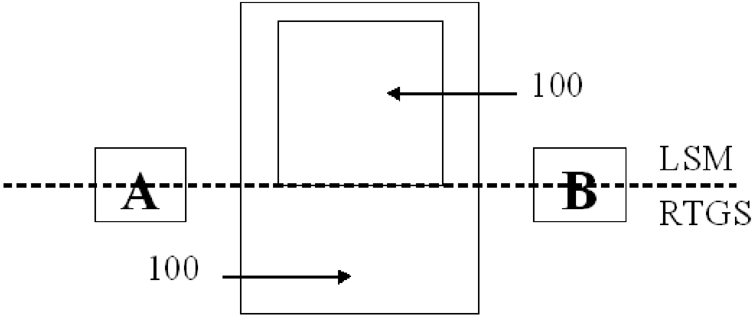

Figure: Alternative LSMs: big box and small box approach.

Problem solved

The bank problem:

$$\min_{L_0} \underset{\lambda,\theta}{E} \left[\min_{P,Q} \underset{\phi(\omega)}{E} (C1 + C2) \right]$$

s.t.

$$\begin{aligned} C_1 &= (1-Q)\left[PI(L_1<\mu)(1-\omega^i)(R+\gamma)+(1-P)\gamma\right] \\ &\quad +Q(1-P)(1-\omega^q)\gamma+\kappa L_0 \end{aligned}$$

$$\begin{aligned} C_2 &= \left\{(1-Q)(1-\omega^i)\left[(1-P)+PI(L_1<\mu)\right]+Q(1-P)(1-\omega^q)\right\} \\ &\quad \times \max(\mu - L_1, 0)\Gamma \end{aligned}$$

Equilibrium

A strategy $\{L_0^*, P^*(\lambda,\gamma;L_0), Q^*(\lambda,\gamma;L_0)\}$ is a symmetric subgame perfect Nash equilibrium strategy, if there exists a set of beliefs $\omega = \{\omega^s, \omega^i, \omega^q\}$ such that:

$$\begin{aligned}
\left\{ \begin{array}{l} P^*(\lambda,\gamma;L_0) \\ Q^*(\lambda,\gamma;L_0) \end{array} \right\} &= \arg\min_{P,Q} C(L_0, P(\lambda,\gamma;L_0), Q(\lambda,\gamma;L_0), \omega) \ \forall\, \lambda, \gamma, L_0 \\
L_0^* &= \arg\min_{L_0} \mathop{E}_{\lambda,\gamma} \left[C(L_0, P^*(\lambda,\gamma;L_0), \omega) \right] \\
\omega &= \Omega(L_0^*, \omega)
\end{aligned}$$

Optimal payment behavior in the morning

P13 If $L_1 \geq \mu$, then banks choose to pay early, unless $\omega^q = 1$, in which case they queue.

P14 If $L_1 < \mu$, then banks find it optimal to queue.

Equilibrium probability to receive payments

$n = 2$: $\Omega^s = \tau_s + \tau_i + \tau_q$

$n \to \infty$: $\Omega^s = \frac{\tau_s}{\tau_s + \tau_d}$

$n = 2$: $\Omega^q = \tau_s + \tau_q$

$n \to \infty$: $\Omega^q = \frac{\tau_s}{\tau_s + \tau_d}$

$n = 2$: $\Omega^i = \tau_s$

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$$\Gamma = \frac{(\tau_d + \tau_i + \tau_q)^{n-1}}{n} \Psi < \Psi.$$

Optimal collateral choice

L_0^*

With LSM the equilibrium strategy is $L_0^* = 1 - \mu$, $P^*(\lambda, \gamma, L_0) = 0$, $Q^*(\lambda, \gamma, L_0) = 1 \;\; \forall \lambda, \gamma$ and $\omega^q = 1$.

Social planner solution

Without LSM:

$L_0^* = 1 - \mu$ and $P^*(\lambda, \gamma, L_0) = 0$, $\omega^i = 0 \ \forall \lambda, \gamma$ if $(3\mu - 2)\kappa > \gamma\theta$,

otherwise $L_0^* = 2\mu - 1$ and $P^*(\lambda, \gamma, L_0) = 1$, $\omega^i = 1 \ \forall \lambda, \gamma$.

With LSM:

$L_0^* = 1 - \mu$, $P^*(\lambda, \gamma, L_0) = 0$, $Q^*(\lambda, \gamma, L_0) = 1$, $\omega^q = 1 \ \forall \lambda, \gamma$.

CHAPS

Calibrate $1-\mu$ and π for CHAPS:

- Size of liquidity shock: $1-\mu = 0.062$
- Probability of the shock: $\overline{\pi} = 0.24$
- The current level of collateral is at $L_0 = 0.14$

Thus introduction of LSM would lead to about 50% of collateral savings (upper bound).

Key results

- Equilibrium and planner's allocation can differ in collateral-based RTGS without LSM:
  - Too much delay in equilibrium
    if equilibrium allocation $\neq$ planner's.
- For some parameters equilibrium allocation $\equiv$ planner's:
  - All banks delay if collateral cost is high
    (in equilibrium and planner's allocation).
- LSMs *always* welfare improving in collateral-based RTGS, in contrast to Atalay et al. (2008):
  - Introduction of LSM in CHAPS would lead to collateral savings of up to 50%.
  - BoE is implementing queueing algorithm in CHAPS.