

PAST
ECONOMETRICS
EXAMS

Econ 411: Econometrics II

Fall 1999 : Midterm
Maximum Points: 90
Time Allowed: 75 min.

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Figures in parentheses show points allocated to different parts of this test.

1. Here are the OLS estimates of two alternative regressions based on 22 observations:

Model 1: $\hat{Y}_i = 50 + 0.2X_{2i} + 0.5X_{3i} - 2.0X_{4i}; R^2 = 0.80$

and

Model 2: $\hat{Y}_i = 100 + 0.3X_{2i} + 0.4X_{3i}; R^2 = 0.76.$

- (a) Use the F test at the 5% level of significance to select between the alternative models. (10 pts)
- (b) Which model would you choose by the \bar{R}^2 criterion? Show numerical calculations to support your answer. (5 pts)
- (c) Construct a 95% confidence interval for the coefficient of X_4 in Model 1. (10 pts)

2. Consider the 2-variable regression without intercept:

$$Y_i = \beta X_i + u_i$$

where u_i satisfies all the assumptions of the classical linear regression.

Now consider two alternative estimators:

$$\hat{\beta} = \frac{\sum X_i Y_i}{\sum X_i^2} \text{ and } \beta^* = \frac{\sum Y_i}{\sum X_i}$$

- (a) Show that both estimators are unbiased. (10 pts)
- (b) Obtain the variance of each estimator and show that the variance of $\hat{\beta}$ is lower than the variance of β^* . (15 pts)
- (c) Suppose that one erroneously included an intercept and estimated a 2-variable regression even though the true model does not include a constant. Will the OLS estimator of β be unbiased in this case? (10 pts)
- (d) Now suppose that the true model did have an intercept and is

$$Y_i = \alpha + \beta X_i + u_i$$

but the model was estimated without an intercept.

Will the estimator of β be unbiased in this case? Explain. (10 pts)

3(a) The regression model

$$Y_i = \beta_1 + \beta_2 X_i + u_i \text{ applies for females}$$

and the model

$$Y_i = \alpha_1 + \beta_2 X_i + u_i \text{ applies for males.}$$

Here we assume that the disturbance term u_i has the same distribution for both groups.

Now consider two dummy variables

$$M_i = 0 \text{ for females and}$$

$$= 1 \text{ for males;}$$

$$F_i = 1 \text{ for females and}$$

$$= 0 \text{ for males.}$$

Two alternative models are proposed combining both regressions:

$$\text{Model 1: } Y_i = \beta_1 + \delta M_i + \beta_2 X_i + u_i$$

$$\text{Model 2: } Y_i = \lambda_1 F_i + \lambda_2 M_i + \beta_2 X_i + u_i.$$

What is the relation between the parameters of Model 1 and Model 2? (5 pts)

3(b) Consider the 2-variable regression

$$Y_t = \phi_1 + \beta_2 X_t + u_t \quad (t = 1, 2, \dots, n),$$

where Y_t is the output and X_t is the input in period t . You believe that the output was unusually low in period $t = n$ due to a transportation strike which severely disrupted the supply of input on time. This is captured by including a dummy variable $D_t = 1$ for $t = n$ and 0 otherwise.

The extended model is

$$Y_t = \phi_1 + \delta D_t + \beta_2 X_t + u_t \quad (t = 1, 2, \dots, n).$$

One could either estimate this 3-variable regression using all the data or estimate the original 2-variable regression without the observation for $t = n$.

Show that the estimated coefficient of X_t in the 3-variable regression based on all observations is the same as the coefficient of X_t obtained from the 2-variable regression without the dummy variable but at the same time deleting the observation for $t = n$. (15 pts)