Econ 411: Econometrics II
Spring 1994: Midterm
Maximum Points: 75
Time Allowed: 75 min.

39/75

Figures in parentheses show points allocated to different parts of this test.

1. Consider this game. The following 5 values of the variable x have been selected:

x_1	x_2	x_3	x_4	x_5
4	2	3	1	5

The values of x are known to both me and you. Thereafter, I select two constants (α, β). These constants are known to me but unknown to you. Next I draw 5 random numbers u_i $(i = 1, 2, .., 5)$ from the standard normal distribution $N(0, 1)$. These random numbers drawn are not revealed to you although you know that they have been drawn from $N(0, 1)$. Finally, I construct the random variable

$$Y_i = \alpha + \beta x_i + u_i \quad (i = 1, 2, .., 5).$$

The values of x are already known to you and the corresponding values of Y will be eventually provided to you for estimating (α, β) using least squares. You have two options. You may decide to estimate either $t_1 = \alpha + \beta$ or $t_2 = \alpha - \beta$. You will receive a prize equal to

$$\Pi = 10 - (\theta - t)^2$$

where t is either t_1 or t_2 depending on your choice and θ is the corresponding estimate of t. You must make a decision about what you would like to estimate.

a) Assuming that you want to maximize expected prize, what would be your optimal choice? (10)

b) What would be your expected amount of the prize? (5)

2. Consider the 2-variable regression through the origin:

$$Y_i = \beta X_i + u_i \qquad (1)$$

where the disturbance term has the following stochastic properties:

$$E(u_i) = 0 \ ; \ \ E(u_i^2) = \sigma_u^2 : E(u_i u_j) = 0 \ (i \neq j)$$

and the X_i s are non-stochastic.

a) Obtain the OLS estimator of β $(\hat{\beta})$ and its variance. (8)

b) Now suppose that the model was erroneously specified as:

$$Y_i = \alpha + \beta X_i + u_i. \qquad (2)$$

What is the OLS estimator of β $(\tilde{\beta})$ in this case? (6)

c) Is $\tilde{\beta}$ unbiased? (4)

d) Show that $\tilde{\beta}$ obtained in part (b) will generally have a higher variance than $\hat{\beta}$ obtained in part (a). When will the variances be equal? (6)

e) Now suppose that the true model was (2) but model (1) was estimated by mistake. Will the OLS estimator of β be unbiased in that case? Explain. (6)

3. Consider the vector

$$\iota = \begin{bmatrix} 1 \\ 1 \\ . \\ . \\ 1 \end{bmatrix}$$

and define the matrix

$$M_0 = I - \frac{1}{n}\iota\iota'.$$

a) Show that M_0 is a symmetric idempotent matrix. (5)

b) Consider the regression

$$y_i = \alpha + u_i.$$

Show that the OLS residual for this model is

$e = M_0 y.$ (5)

c) For the multiple regression model

$$y = X\beta + u$$

the OLS estimator $\hat{\beta} = (X'X)^{-1}X'y$ cannot be estimated when $X'X$ is singular. In ridge regression one estimates β as

$$\beta_R = (X'X + kI)^{-1}X'y,$$

where k is a small number added to the diagonal elements of $X'X$ to avoid singularity.

Is β_R unbiased? (5)

4. a) In the two variable regression

$$y_t = \alpha + \beta X_t + u_t,$$

$\hat{\beta} = 2.0$ with standard error equal to 0.50. The sample size was 12. What was the R^2 of the fitted model? (6)

b) Consider the regressions

$$Y_t = \beta_1 + \beta_2 X_{2t} + u_t$$

and

$$Y_t = \alpha_1 + \alpha_2 X_{2t} + \alpha_3 X_{3t} + u_t \ .$$

Show that if X_2 and X_3 are orthogonal, $\hat{\beta}_2 = \hat{\alpha}_2$. (6)

c) Suppose that $\hat{\beta}$ is the OLS estimator of β in the regression

$$y = X\beta + u$$

and δ is any other estimator.

Show that

$(y - X\delta)'(y - X\delta) - (y - X\hat{\beta})'(y - X\hat{\beta})$ is non-negative. (3)