Econ 311
Spring 1991

Econometrics I
Mid-term

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Time Allowed : 90 minutes
Maximum Points: 100

Answer all questions. Show intermediate steps and calculations wherever relevant. Figures in parantheses show points assigned for each part.

1. Consider the regression model:

$$Y_i = \beta_1 + \beta_2 X_i + e_i$$

where $E(e_i)=0, var(e_i)=\sigma^2,$ and $cov(e_i, e_j)=0 \quad (i \neq j)$.

a) Derive the OLS estimator $(\hat{\beta}_2)$. (5)

b) Now define the variables

$$Z_i = \frac{Y_i - a}{b} \quad \text{and} \quad W_i = \frac{X_i - c}{d}.$$

Consider the regression

$$Z_i = \alpha_1 + \alpha_2 W_i + \mu_i.$$

What is the relation between the OLS estimator of α_2 $(\hat{\alpha}_2)$ and $\hat{\beta}_2$? (10)

c) Prove that regressions in parts (a) and (b) will have the same R^2. (5)

d) How does the estimated variance of μ_i compare with the estimated variance of e_i ? (5)

2. The demand for Ceylonese tea in the U.S. is given by the equation:

$$\ln Q = \beta_0 + \beta_1 \ln P_C + \beta_2 \ln P_I + \beta_3 \ln P_B + \beta_4 \ln Y + u$$

where Q = import of Ceylonese tea in the U.S;

P_C = price of Ceylone tea;

P_I = price of Indian tea;

P_B = price of Brazilian coffee;

Y = disposable income in the U.S.

M_C = $ value of Ceylone tea imported in the U.S.

The following regressions were obtained from n = 22 observations.

$$\ln Q = \underset{(2.0)}{2.837} - \underset{(0.987)}{1.481} \ln P_C + \underset{(0.690)}{1.181} \ln P_I + \underset{(0.134)}{0.186} \ln P_B + \underset{(0.370)}{0.257} \ln Y \qquad ESS = 0.4277.$$

U

$$\ln M_C = - \underset{(0.820)}{0.738} + \underset{(0.155)}{0.199} \ln P_B + \underset{(0.165)}{0.261} \ln Y$$

$$ESS = 0.6788.$$

R

Figures in paranthese are standard errors.

(a) Explain how one can test

H_0: $\beta_1 = -1$; $\beta_2 = 0, \beta_3, \beta_4 \neq 0$; against

H_1: $\beta_i \neq 0$ $(i = 1, 2, 3, 4)$.

using the information provided. (5)

(b) Use the F test at the 5% level of significance to test the null hypothesis in part (a). (5)

(c) What is the economic significance of the results? (5)

3. Consider the following regression without an intercept:

$$Y_i = \beta X_i + e_i$$

where $E(ee') = \sigma^2 I$.

a) Derive the OLS estimator for β $(\hat{\beta})$. (5)

b) Consider an alternative estimator

$$\bar{\beta} = \frac{\sum_i Y_i}{\sum_i X_i}.$$

Prove that $\bar{\beta}$ is an unbiased estimator. (5)

c) Derive the varainces of $\hat{\beta}$ and $\bar{\beta}$. (8)

(d) Prove that $var(\beta) < var(\bar{\beta})$. (7)

4. Consider the k-variable regression

$$y = X\beta + e$$

$$E(e) = 0;\ E(ee') = \sigma^2 I.$$

Now consider the OLS residuals

$$\hat{e} = y - X\hat{\beta}$$

where $\hat{\beta} = (X'X)^{-1}X'y$.

$I - X(X'X)^{-1}X'$

$\frac{1}{n-k}$

$\hat{e} = y - X\beta$

$\hat{e} = y - X(X'X)^{-1}X'y = My \quad E(\hat{e}) \Rightarrow M(X\beta + e) = Me$

a) Prove that $E(\hat{e}) = 0$. (5)

$\hat{e} = Me$

$\hat{e}' = e'M$

$\hat{e}\hat{e}' = Mee'M$

b) Derive $\boxed{E(\hat{e}\hat{e}')}$ and show that $E(\hat{e}\hat{e}') \neq \sigma^2 I$. (10)

$E(\hat{e}\hat{e}') = E(Mee'$

$= E(Tr\, ee'M$

c) Consider now the regression

$$\hat{e} = X\hat{\delta} + \eta.$$

$\sigma Tr(M)$

$\Rightarrow \sigma^2(n-k)$

Prove that the R^2 for this regression will always be 0. (5)

5. The following summary data relate to n = 5 observations:

$$X'X = \begin{bmatrix} 5 & 15 & 25 \\ 15 & 55 & 81 \\ 25 & 81 & 189 \end{bmatrix} \quad (X'X)^{-1} = \begin{bmatrix} 26.7 & 4.5 & -8.0 \\ 4.5 & 1.0 & -1.5 \\ -8.0 & -1.5 & 2.5 \end{bmatrix} \quad X'y = \begin{bmatrix} 20 \\ 76 \\ 109 \end{bmatrix}$$

var(β_1)

var(β_3)

The data provided above were used to estimate the regression:

$$Y_i = \beta_1 + \beta_2 X_{i2} + \beta_3 X_{i3} + e_i$$

where each e_i is independently distributed as $N(0, \sigma^2)$.

The OLS estimators were :

$$\hat{\beta}_1 = 4;\ \hat{\beta}_2 = 2.5;\ \hat{\beta}_3 = -1.5;\ \hat{\sigma}^2 = 13.25.$$

a) Test at the 10% level of significance the hypothesis $H_0: \beta_2 = 2$ using the "t" test. (5)

b) Test at the 5% level of significance the hypothesis $H_0: \beta_2 + \beta_3 = 0$ using the F test. (10)