Econ 411
Fall 2002

Econometrics II
Midterm Exam
Maximum Points:

Subhash Ray

Answer 1 question from Group A and both questions from Group B.

Group A:

1. Consider the 2-variable regression model

$$Y_i = \beta_1 + \beta_2 X_{2i} + u_i$$

where $E(u_i) = 0$, $E(u_i^2) = \sigma^2$, and $E(u_i u_j) = 0$ $(i \neq j)$.

The n observations in the sample are from two groups: male and female. The first n_1 observations are for males and the remaining $n_2 = n - n_1$ observations are for females. The variable X_2 is a binary variable that takes the value 0 whenever the observation relates to a male and the value 1 whenever the observation relates to a female. Suppose that $\overline{Y}_1$ is the sample mean of Y for males and $\overline{Y}_2$ is the sample mean for females.

(a) Derive the OLS estimators $(\hat{\beta}_1, \hat{\beta}_2)$.

(b) Derive the variance of $\hat{\beta}_2$.

(c) Now consider, for the same sample, the model

$$Y_i = \alpha_1 X_{1i} + \alpha_2 X_{2i} + u_i$$

where X_1 takes the value 1 for males and 0 for females and X_2 is as described earlier. This model does not include an intercept. Show that the OLS estimators $(\hat{\alpha}_1, \hat{\alpha}_2)$ are the same as the estimators $(\hat{\beta}_1, \hat{\beta}_2)$.

2. Consider the 2-variable regression without intercept:

$$Y_i = \beta X_i + u_i$$

where u_i satisfies all the assumptions of the classical linear regression.

Now consider two alternative estimators:

$$\hat{\beta} = \frac{\sum X_i Y_i}{\sum X_i^2} \text{ and } \beta^* = \frac{\sum Y_i}{\sum X_i}.$$

(a) Show that both estimators are unbiased. (10 pts)

(b) Obtain the variance of each estimator and show that the variance of $\hat{\beta}$ is lower than the variance of β^*.

(c) Now suppose that the true model did have an intercept and is

$$Y_i = \alpha + \beta X_i + u_i$$

but the model was estimated without an intercept.

Will the estimator of β be unbiased in this case? Explain.

Group B:

3. Consider the regression model

$$y = X\beta + u$$

where $\beta = (\beta_1, \beta_2)$.

The following information relatc to 2 independent samples of size 20:

sample 1: $X'X = \begin{bmatrix} 20 & 100 \\ 100 & 600 \end{bmatrix}; X'y = \begin{bmatrix} 500 \\ 2700 \end{bmatrix}$

sample 2: $X'X = \begin{bmatrix} 20 & 200 \\ 200 & 2400 \end{bmatrix}; X'y = \begin{bmatrix} 700 \\ 8000 \end{bmatrix}$

(a) Obtain the OLS estimates of β_1 and β_2 from sample 1.

(b) Obtain the OLS estimates of β_1 and β_2 from sample 2.

(c) Now combine the two samples to obtain the OLS estimates of β_1 and β_2.

(d) Compute the variance of $\hat{\beta}_2$ obtained from the individual samples and the pooled sample.

4. (a) The following relate to the estimated regression model:

$$\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i :$$

$\hat{\alpha} = 0.4; \hat{\beta} = 1.0; \text{var}(\hat{\beta}) = 0.01; \text{n} = 52.$

Find the R^2 of the model.

(b) The linear regression model applies to

$$E(y) = X\beta$$

with $X'X = \begin{bmatrix} 4 & 2 \\ 2 & 5 \end{bmatrix}$.

A sample of 32 observations yields $(\hat{\beta}_1 = 2, \hat{\beta}_2 = 5)$ and $e'e = 60$.

Test the hypothesis

$H_0 : \beta_1 = \beta_2 - 1$

at the 5% level of significance.

(c) Here are the OLS estimates of two alternative regressions based on 22 observations:

Model 1: $\hat{Y}_i = 50 + 0.2X_{2i} + 0.5X_{3i} - 2.0X_{4i}; R^2 = 0.80$

and

Model 2: $\hat{Y}_i = 100 + 0.3X_{2i} + 0.4X_{3i}; R^2 = 0.76.$

Use the F test at the 5% level of significance to select between the alternative models.

(d) Which of the 2 models reported in part (c) would you choose using the $\overline{R}$ criterion? Show the necessary calculations.